

Green-Schwarz superstring from type IIB matrix model

Yoshihisa Kitazawa^{1,2,*} and Satoshi Nagaoka^{1,†}

¹*High Energy Accelerator Research Organization (KEK), Tsukuba, Ibaraki 305-0801, Japan*

²*Department of Particle and Nuclear Physics The Graduate University for Advanced Studies Tsukuba Ibaraki 305-0801 Japan*

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We construct Green-Schwarz (GS) light-cone closed superstring theory from type IIB matrix model. A GS light-cone string action is derived from two dimensional $\mathcal{N} = 8$ $U(n)$ noncommutative Yang-Mills (NCYM) by identifying noncommutative scale with string scale. Supersymmetry transformation for the light-cone gauge action is also derived from supersymmetry transformation for IIB matrix model. By identifying the physical states and interaction vertices, string theory is perturbatively reproduced.

I. INTRODUCTION

While the theory of general relativity classically gives a proper description of spacetime, it brings unrenormalizable divergences by quantum effects. In order to avoid the divergence of quantum gravity, we need to modify the geometry at short length scale. Technically, we need to introduce an effective ‘cut-off’ in the theory. Superstring theory naturally gives such a cut-off, since the ultra-violet divergence is removed by the cut-off at string scale α' . While string theory is well-defined only in ten dimensions, the structure of space-time, which includes the number of effectively emergent space-time dimensions, should be decided by itself. Type IIB matrix model is proposed as a nonperturbative formulation of superstring theory [1]. Originally, it is derived by a matrix regularization of Green-Schwarz type IIB superstring. Light-cone superstring field theory of type IIB superstring is reproduced from a continuum limit of loop equations for Wilson loops in the large N limit [2]. Since this model is in some sense a Lorentz covariantly regularized theory, not only space, but also even time may naturally emerge in this model.

In string theory, the effect of cut-off is seen through the T-duality transformation [3, 4]. In a compactified space with the radius R , a string whose compactification scale R is equivalent to that of α'/R . In type IIB matrix model, a similar kind of equivalence is also seen. Although there is no dimensionful parameter in IIB matrix model, once we select a certain background, then, the characteristic dimensionful parameter, which is called the noncommutative parameter, emerges. It is proposed that the self-dual scale of this theory exists at a noncommutative scale θ [5, 6, 7]. Another important property of type IIB matrix model is supersymmetries. In string theory, interaction vertex for GS light-cone closed superstring is completely determined by the supersymmetry transformation. In IIB matrix model, the coupling to supergrav-

ity multiplet is determined by the supersymmetry transformation [8]. The action of IIB matrix model can be derived by the dimensional reduction of ten dimensional super Yang-Mills (SYM) theory to zero dimension. Each SYM with different dimensionality shows an interesting behavior and they are related with each other in a large variety of way. In particular, two dimensional $\mathcal{N} = 8$ $SU(N)$ SYM is proposed as a type IIA multi-string theory [9]. It is conjectured as another candidate of non-perturbative formulation of superstring theory.

In this paper, we search for the description of perturbative superstring theory from type IIB matrix model. Type IIB matrix model naturally includes $\mathcal{N} = 8$ noncommutative Yang-Mills theory (NCYM) [10, 11, 12]. In the commutative limit, it reduces to ordinary gauge theory. In such a sense, type IIB matrix model contains matrix string theory in the large N limit. In order to clarify and develop such a perspective, we derive a Green-Schwarz (GS) superstring action from type IIB matrix model. We can formulate a string perturbation theory and calculate multi-point correlation functions from $U(n)$ NCYM since multiple string worldsheets are derived from $U(n)$ NCYM. Furthermore, we derive supersymmetry transformation for the GS light-cone superstring action from type IIB matrix model. The massless particle vertex operators for GS light-cone superstring theory are determined by the requirement that they transform into one another under supersymmetry transformations. Since we reproduce the supersymmetry transformation for light-cone GS superstring from type IIB matrix model, we can construct the corresponding vertex operators in the same procedure.

II. SUPERSTRING FROM TYPE IIB MATRIX MODEL

A. Superstring action

Type IIB matrix model is defined by the action as

$$S = -\frac{1}{g^2} \text{Tr} \left(\frac{1}{4} [A^\mu, A^\nu] [A_\mu, A_\nu] + \frac{1}{2} \bar{\psi} \Gamma^\mu [A_\mu, \psi] \right) \quad (\text{II.1})$$

*Electronic address: kitazawa@post.kek.jp

†Electronic address: nagaoka@post.kek.jp

where ψ is a ten dimensional Majorana-Weyl spinor. $A_\mu (\mu = 0, 1, \dots, 9)$ and ψ are $N \times N$ Hermitian matrices. The coupling constant g can be absorbed by the field redefinition

$$\begin{aligned} A_\mu &\rightarrow g^{1/2} A_\mu, \\ \psi &\rightarrow g^{3/4} \psi. \end{aligned} \quad (\text{II.2})$$

d dimensional Euclidean NCYM are obtained by expanding this action around d dimensional flat background:

$$[p_\mu, p_\nu] = i\theta_{\mu\nu}. \quad (\text{II.3})$$

In this construction, two dimensional NCYM with $\mathcal{N} = 8$ supersymmetry is the lowest dimensional theory and the lagrangian is written as

$$\begin{aligned} S = -\frac{\theta}{8\pi g^2} \int d^2 x \text{tr} & \left([D^{\tilde{\mu}}, D^{\tilde{\nu}}][D_{\tilde{\mu}}, D_{\tilde{\nu}}] + 2[D^{\tilde{\mu}}, \phi^i][D_{\tilde{\mu}}, \phi_i] \right. \\ & \left. + [\phi_i, \phi_j][\phi_i, \phi_j] + 2\bar{\psi}\Gamma^{\tilde{\mu}}[D_{\tilde{\mu}}, \psi] + 2\bar{\psi}\Gamma_i[\phi_i, \psi] \right)_*, \end{aligned} \quad (\text{II.4})$$

where $\tilde{\mu}, \tilde{\nu} = 0, 1$ and $i, j = 2, \dots, 9$. $*$ product is described by

$$a * b = \exp\left(\frac{iC^{\mu\nu}}{2} \frac{\partial^2}{\partial \xi^\mu \partial \eta^\nu}\right) a(x + \xi) b(x + \eta)|_{\xi=\eta=0} \quad (\text{II.5})$$

$D_{\tilde{\mu}}$ are covariant derivative operators which contain the gauge field as

$$[D_{\tilde{\mu}}, \hat{o}] = [\hat{p}_{\tilde{\mu}} + \hat{a}_{\tilde{\mu}}, \hat{o}] \rightarrow \frac{1}{i} \partial_{\tilde{\mu}} \hat{o} + a_{\tilde{\mu}} * \hat{o} - \hat{o} * a_{\tilde{\mu}}(x) \quad (\text{II.6})$$

Trace of the matrices maps into the integral of the functions as

$$\text{Tr} \rightarrow \frac{\theta}{2\pi} \text{tr} \int d^2 x. \quad (\text{II.7})$$

Remaining trace in (II.4) is the trace over $U(n)$ gauge group in two dimension. Thus, two dimensional theory is constructed from type IIB matrix model.

We will identify the perturbative string spectrum in the IR limit of two dimensional NCYM. First of all,

i) $*$ product goes to ordinary commutative product since higher derivatives in the product can be neglected. The action (II.4) becomes the commutative $\mathcal{N} = 8$ $U(n)$ super Yang-Mills in this limit

$$\begin{aligned} S = -\frac{\theta}{8\pi g^2} \int d^2 x \text{tr} & \left(F_{\tilde{\mu}\tilde{\nu}}^2 + 2(D_{\tilde{\mu}}\phi_i)^2 + [\phi_i, \phi_j][\phi_i, \phi_j] \right. \\ & \left. + 2\bar{\psi}\Gamma^{\tilde{\mu}}D_{\tilde{\mu}}\psi + 2\bar{\psi}\Gamma_i[\phi_i, \psi] \right). \end{aligned} \quad (\text{II.8})$$

This action includes 8 matrix scalar fields ϕ_i and 16 matrix spinor fields $\psi = (s^a, \dot{s}^a)$. These fields transform in 8_v , 8_c and 8_s representations of $SO(8)$ group. The perturbative vacua of this action are represented by the

diagonal matrices $\phi_i = (\phi_{\text{diag}})_i$, which form the moduli space of this theory.

By assuming that all the eigenvalues of matrices do not coincide with each other at any points on the worldsheet, all the excitations of off-diagonal modes become massive. Then,

ii) only diagonal elements are relevant in the low energy limit since massless excitations come from diagonal elements. The contribution of the terms $[\phi_i, \phi_j][\phi_i, \phi_j]$ and $2\bar{\psi}\Gamma_i[\phi_i, \psi]$ vanish since diagonal terms commute. Gauge fields on two dimension, which come from the first term in (II.8), decouple from other fields.

In order to identify the action (II.8) with a light-cone superstring action, we map the worldsheet coordinate system from R^2 coordinate into $R^1 \times S^1$ coordinate as

$$z \equiv x_0 + ix_1 = e^{\tau + i\sigma}. \quad (\text{II.9})$$

The origin of the x coordinate is the special point where the vertex operators are inserted. By the rescaling,

$$\psi_R \rightarrow \frac{1}{\sqrt{z}} \psi_R, \quad \psi_L \rightarrow \frac{1}{\sqrt{z}} \psi_L, \quad (\text{II.10})$$

we obtain the action for a single string with the winding number w

$$\begin{aligned} S = -\frac{\theta}{4\pi g^2} \int_0^\infty d\tau \int_0^{2\pi w} d\sigma & \left((\partial_\tau \phi_i)^2 + (\partial_\sigma \phi_i)^2 \right. \\ & \left. + \bar{\psi}(\Gamma^+ \partial_+ + \Gamma^- \partial_-) \psi \right) \end{aligned} \quad (\text{II.11})$$

Multiple strings are obtained in general. Thus, two dimensional NCYM (II.4) reduces to the GS light-cone superstring theory in the IR limit. A string with the winding number w along the σ direction is described in Figure 1. We have identified $\phi_i(\sigma) = \phi_i(\sigma + 2\pi w)$ in this case.

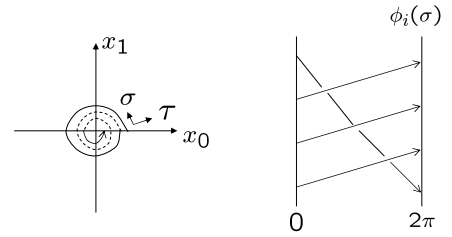


FIG. 1: A string winds w times along σ direction.

θ is introduced as a characteristic scale of background (II.3). In this background, θ is a unique dimensionful parameter. GS light-cone superstring action is obtained by identifying $\frac{\theta}{4\pi g^2} \equiv \frac{1}{4\pi\alpha'}$. Since (II.11) is obtained by taking the commutative limit, the winding number w has to be large. We can reproduce the Veneziano type formula if we sum up the contribution from infinite towers of the spectrum. On-shell condition is relevant only to external modes. In fact, we calculate four point amplitude from type IIB matrix model where stringy effect is relevant [13].

The winding number w along the angular direction can be reinterpreted as light-cone momenta p^+ in a T-dual interpretation. In this interpretation, we obtain type IIA superstring theory. Indeed, vertex operators in GS light-cone superstring for type IIA supergravity multiplet are constructed [13] from original vertex operators for type IIB matrix model [8, 14, 15]. Vertex operators in matrix model are utilized to show the localization of gravity on D-brane [16] like a Randall-Sundrum model [17].

The duality relation is shown in Figure 2. We show our construction along with Dijkgraaf-Verlinde-Verlinde's matrix string theory. Two dimensional NCYM obtained here is the action of a D-string. In DVV's matrix string theory, two dimensional SYM action, which is obtained by the compactification along 9-direction (T-duality transformation) from BFSS matrix model, is the action of D-strings. The former is uncompactified theory, although the latter is compactified along 9-direction, which is the direction longitudinal to the worldvolume of the D-string. They obtain type IIA string in a T-dual interpretation along the light-cone direction. On the other hand, we obtain type IIA string by radial quantization.

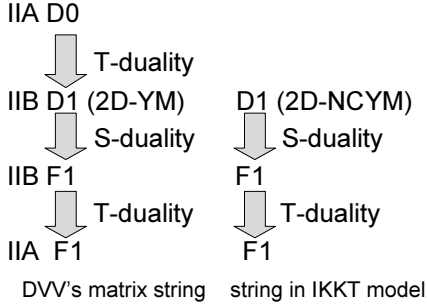


FIG. 2: Duality relation between type IIA and type IIB string is shown.

B. Supersymmetry transformation

$\mathcal{N} = 2$ supersymmetry transformation in IIB matrix model is written as

$$\begin{aligned}\delta^{(1)}\psi &= \frac{i}{2}[A_\mu, A_\nu]\Gamma^{\mu\nu}\epsilon, \\ \delta^{(1)}A_\mu &= i\bar{\epsilon}\Gamma_\mu\psi, \\ \delta^{(2)}\psi &= -\eta, \\ \delta^{(2)}A_\mu &= 0.\end{aligned}\tag{II.12}$$

In the two dimensional NC background, this transformation can be written as 8 scalar fields, 16 spinor fields and

2 gauge fields. In the low energy limit, it reduces to

$$\begin{aligned}\delta^{(1)}s_a &= -\dot{\phi}^i\gamma_{a\dot{a}}^i\epsilon^{\dot{a}}, & \delta^{(1)}s_{\dot{a}} &= -\dot{\phi}^i\gamma_{\dot{a}a}^i\epsilon^a, \\ \delta^{(1)}\phi_i &= 2(\bar{\epsilon}^{\dot{a}}\gamma_{a\dot{a}}^i s^a + \bar{\epsilon}^a\gamma_{\dot{a}a}^i s^{\dot{a}}), \\ \delta^{(2)}s_a &= -\eta^a, & \delta^{(2)}s_{\dot{a}} &= -\eta^{\dot{a}}, \\ \delta^{(2)}\phi_i &= 0,\end{aligned}\tag{II.13}$$

where we have redefined $\eta^a \rightarrow \eta^a + \theta\epsilon$, $\eta^{\dot{a}} \rightarrow \eta^{\dot{a}} - \theta\epsilon$ to absorb the constant shift. $\gamma_{a\dot{a}}^i$ are Clebsch-Gordan coefficients for coupling three inequivalent $\text{SO}(8)$ representations. Indeed, these transformations leave the Green-Schwarz light-cone string action (II.11) invariant.

The sixteen supersymmetry charges are 8_s and 8_c given by

$$\begin{aligned}Q^a &= s_0^a, \\ Q^{\dot{a}} &= \sqrt{2}\gamma_{\dot{a}a}^i \sum_{-\infty}^{\infty} s_{-n}^a \alpha_n^i,\end{aligned}\tag{II.14}$$

for the left mover and

$$\begin{aligned}\tilde{Q}^a &= s_0^{\dot{a}}, \\ \tilde{Q}^{\dot{a}} &= \sqrt{2}\gamma_{\dot{a}a}^i \sum_{-\infty}^{\infty} s_{-n}^{\dot{a}} \tilde{\alpha}_n^i,\end{aligned}\tag{II.15}$$

for the right mover where quantization of superstring action involves 4 sets of modes denoted by α_n^i , $\tilde{\alpha}_n^i$, s_n^a and $s_n^{\dot{a}}$. They satisfy the relations

$$\begin{aligned}[\alpha_m^i, \alpha_n^j] &= m\delta_{m+n}\delta^{ij}, \\ [\tilde{\alpha}_m^i, \tilde{\alpha}_n^j] &= m\delta_{m+n}\delta^{ij}, \\ \{s_m^a, s_n^b\} &= \delta_{m+n}\delta^{ab}, \\ \{s_m^{\dot{a}}, s_n^{\dot{b}}\} &= \delta_{m+n}\delta^{\dot{a}\dot{b}}.\end{aligned}\tag{II.16}$$

Let us consider the left moving bosonic (vector) and fermionic (spinor) vertex operators

$$\begin{aligned}V_B(\zeta, k) &= \zeta \cdot B e^{ik \cdot \phi}, \\ V_F(u, k) &= u F e^{ik \cdot \phi},\end{aligned}\tag{II.17}$$

where ζ is a polarization vector which represents the wave function for the vector state and u represents the wave function for the spinor states. The coefficients B and F are determined by the supersymmetry transformation

$$\begin{aligned}[\eta^a Q^a, V_F(u, k)] &\approx V_B(\tilde{\zeta}, k), \\ [\eta^a Q^a, V_B(u, k)] &\approx V_F(\tilde{u}, k).\end{aligned}\tag{II.18}$$

The symbol \approx means that equality is only required for on-shell matrix elements. The structure of the vertex operators is uniquely determined by the requirement of global supersymmetry. The closed string vertex operators are constructed by the product of open string vertex operators

$$V(\sigma, \tau) = V_R(\tau - \sigma)V_L(\tau + \sigma),\tag{II.19}$$

where the left moving operator $V_L(\tau - \sigma)$ and the right moving operator $V_R(\tau + \sigma)$ are either bosonic V_B or fermionic V_F . Explicit forms of vertex operators are constructed [13] within the kinematics where external momentum along the light-cone direction is $k^+ = 0$, since A^- has serious ordering problem in the light-cone gauge¹.

III. CONCLUSION

We have derived Green-Schwarz light-cone superstring theory from type IIB matrix model. As illustrating in Figure 3, (1) two dimensional background in type IIB matrix model reduces to Green-Schwarz superstring action in the low energy limit. The field ϕ is interpreted as a displacement of the *type IIA superstring*, since this theory is derived from type IIB matrix model in a *T-dual interpretation*. This is similar to the derivation of matrix string by Dijkgraaf, Verlinde and Verlinde. (2) In order to derive string perturbation from type IIB matrix model, we have directly derived supersymmetry transformation for GS light-cone string from type IIB matrix model. The supersymmetry transformation also shows symmetries of the type IIA superstring theory. Since vertex operators are determined by the requirement that they transform into one another under supersymmetry transformations, we can uniquely determine the vertex operators to calculate the amplitude of multi-point function. On the other hand, the vertex operators in type IIB matrix model are constructed in [8, 14, 15] where external momenta are carried by the Wilson lines. (3) We can derive the vertex operators for GS type IIA light-cone string directly from type IIB matrix model, which will be done in [13].

In this paper, as a first step to describe the string interaction in type IIB matrix model, we have obtained the second quantized worldsheet theory by analyzing $U(n)$ NCYM. Fundamental scale θ and the maximal supersymmetry play crucial roles for the identification. As

¹ In order to resolve this problem, a separate Fock space for each string is introduced.

we have identified the noncommutative scale with string scale, we need to understand its significance more deeply such as the relation with space-time uncertainty principle [18]. If the Wilson line carries the momentum p^+ , it extends $\mathcal{O}(\frac{|p^+|}{\theta}) \sim \mathcal{O}(\alpha'|p^+|)$ in the orthogonal direction. Thus, the length of Wilson line may be identified with light-cone momentum, which is familiar in the light-cone formulation of superstring theory. In order to treat multi-string interactions, we need to construct some new types of formulations like closed superstring field theory. In Dijkgraaf, Verlinde and Verlinde's matrix string theory, they introduce an interaction of multi strings as a recombination of intersecting (D-)strings [19]. It may also be interesting to consider this effect within type IIB matrix model.

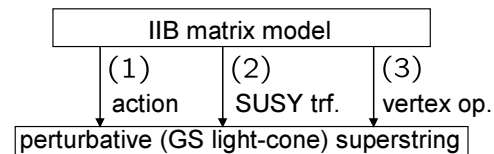


FIG. 3: Green-Schwarz light-cone superstring theory are perturbatively obtained by three ways.

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